DAGM v2.1 — Appendix supplement

Empirical tables, historical dataset construction, supplementary figures, and source notes

Contents

This volume is the **Paper v2.1 appendix supplement**, built from `documentation/Paper_v2.1/appendix/` and stitched into `DAGM_v2.1_appendix_supplement.pdf`. It accompanies the main Paper v2.1 PDFs (`Paper_v2.1_{public,full_draft,internal}.md`). **Numerical authority** for claims in the paper body remains the main text; this supplement collects **extended grids** (Appendix A), the **full historical-dataset construction** (Appendix B), **supplementary figures** (Appendix C, including couple-plan charts in C.3), optional charts (Appendix D), **legacy-sensitivity** (Appendix A.5), **multi-policy legacy comparison** (Appendix A.5b), and the **source manifest** (end matter).

Build from repository root: `python scripts/build_paper_v21_release_pdfs.py` — requires Pandoc and a LaTeX engine (same as the main paper). Regenerate empirical tables: `python scripts/build_paper_v21_release_pdfs.py --refresh-appendix-tables`. Regenerate figures: `python documentation/charts/build_appendix_c_assets.py`.

Appendix A — Empirical supplements

A.1 Validation and reproducibility

This paper evaluates withdrawal rules on **historical rolling-window** simulations: each run is a **single realized return path** through the dataset, not a draw from a resampled or Monte Carlo distribution. The empirical engine is **deterministic** given inputs (dataset row, horizon, allocation, policy parameters, governance settings): the same configuration reproduces the same sequence of withdrawals when re-run on the same codebase.

Protocol (summary). The core matrix in §5.2–5.3 uses **25** Lab configurations (combinations of USD/EUR dataset, horizon, and equity/bond mix) with **rolling start years** and a **one-year step** between starts, subject to dataset coverage. Horizons **10, 15, 20, and 30** years appear where the sample allows; the USD long-horizon block in §5.4 adds **40-** and **50-**year horizons on the same rolling logic. **Five** equity shares from 100/0 to 20/80 are used in the main matrix. **Appendix B** documents the annual return files (`USD_1928_2025`, `EUR_1999_2025`).

Outputs and traceability. Published numbers are read from **Lab / Sandbox batch exports** under `qa_lab/new_epochs/` (post–Manage CPI-fix engine, May 2026): JSON/CSV summaries per epoch folder (`epoch_1 … epoch_6`, `epoch_8` for lifecycle §6, `epoch_11` for legacy , `epoch_12` for path-depth §5.6). **Epoch 3** withdrawal-stability tables use `*_corridor_governed__withdrawal_stability_summary.csv` in `qa_lab/new_epochs/epoch_3/runs/`, produced from each run's `*_yearly_paths.json` by `qa_lab/new_epochs/build_epoch3_stability_from_yearly_paths.py`. **§5.7** glide–flat results come from `qa_lab/new_epochs/epoch_6/runs/`. Pre-fix snapshots: `qa_lab/_archive_pre_manage_cpi_fix/` (audit only).

Aggregation conventions (critical). Unless a section explicitly states otherwise:

- For **level metrics** in §5.2–5.5 (W^{avg}, W^{min}, W^{max} in real terms), configuration-level figures are typically **means across rolling start windows** of **within-window** summaries (mean of window averages, mean of window minima, mean of window maxima). This matches the §5.2 note in the main paper.
- For **§5.7 glide vs. flat**, the paper uses a **different** cross-window rule for W_{min} and W_{max}: **minimum across windows** of each window's minimum, and **maximum across windows** of each window's maximum ("tail across windows"). That convention matches the glide–flat diagnostics in the Lab exports and is **not** interchangeable with the §5.2 level-table convention.

Validation scope. The public paper describes **badge** and **positioning** logic at a high level; exact production rules remain in the product implementation. Empirical **validation** in this document is the **reported historical perfor-**

mance of the engine under the stated batches, not a third-party audit of the commercial UI.

Table A.1 — Empirical protocol (core matrix, §5.2–5.3)

Item	Specification
Datasets	USD 1928–2025; EUR 1999–2025 (see Appendix B)
Rolling step	1 year between start dates (where coverage allows)
Core configurations	25 rolling setups (Epoch 2 / Epoch 3 matrix)
Policies compared	Dynamic ARVA, GK, SWR, CPW; corridor-governed DAGM
Real deflation	Dataset `cpi` column (annual decimal rate); cumulative price index per window

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A.2 Additional tables and robustness diagnostics

The main text (§5) develops the **narrative** for each epoch. This section collects **compact win-count and robustness summaries** so the appendix PDF remains **self-contained** for readers who need tables without returning to the body. Substantive interpretation follows §5.2–5.5; Epoch **5A** (inflation-linked sleeve) and **5B** (gold sleeve) are **qualitative** robustness checks (ordering of policies unchanged)—see §5.5.A–B for full discussion.

Table A.2 — Epoch 2: DAGM vs static references (wins out of 25 configurations)

Metric	DAGM vs ARVA	DAGM vs GK	DAGM vs SWR	DAGM vs CPW
W_{real}^{avg}	18 / 25	25 / 25	25 / 25	25 / 25
W_{real}^{min}	7 / 25	24 / 25	24 / 25	25 / 25
W_{real}^{max}	25 / 25	25 / 25	25 / 25	25 / 25

Table A.3 — Epoch 3: DAGM vs ARVA, downside stability (wins out of 25)

Source: `qa_lab/new_epochs/epoch_3/runs/*_corridor_governed__withdrawal_stability_summary.csv`. Verify: `python documentation/charts/epoch3_paper_wins.py`.

Diagnostic	DAGM better	Interpretation (see §5.3)
Lower cut-year frequency (cut frequency; years with a negative withdrawal change)	25 / 25	Fewer years with downward withdrawal changes
Lower DWII	25 / 25	Lower cumulative downside instability index
Higher avg_abs_change_pct	21 / 25	More total year-to-year movement (not "smoother" in absolute terms)
Lower mean_max_cut_pct (mean worst-cut)	21 / 25	Less severe average worst single-year cut across paths
Lower mean_avg_cut_pct (conditional avg cut)	14 / 25	Smaller average cut size when a cut occurs

Naming. Table A.4 (below) is the Epoch 5C four-asset win-count grid. It is **not Appendix A.4** (bond-heavy USD 20/80 pathology — a separate narrative section later in this supplement).

Table A.4 — Epoch 5C: four-asset block — DAGM vs references (wins out of 12 configurations)

Level metrics use the same window-mean construction as §5.2 (mean of W^{avg}, W^{min}, W^{max} across rolling windows per configuration).

Metric	DAGM vs ARVA	DAGM vs GK	DAGM vs SWR	DAGM vs CPW
W^{avg}_{real}	12	12	12	12
W^{min}_{real}	2	12	12	12
W^{max}_{real}	12	12	12	12

Epoch 4 (horizon extension). USD **40-** and **50**-year horizons (60/40 and 40/60) are reported in **§5.4**, Table 1 and Figure (full level metrics and long-horizon vs 30y baseline). No separate table is duplicated here.

Epoch 6 (glide vs. flat). Full **cell-by-cell** percentage grids are **Table A.5** and **Table A.6** under §A.3; headline means appear in **§5.7**, Table 2 of the main paper. Supplementary **figures** for glide and twins are in **Appendix C** of this supplement.

Epoch 11 (legacy). See **Appendix A.5** (`A5_legacy_lambda_sensitivity.md`) — life-style vs. bequest sensitivity; cited in **§8.1**. Multi-policy comparison tables: **Appendix A.5b** (`A5b_legacy_policy_compare.md`); cited from **§7**.

Bond-heavy USD pathology. See **Appendix A.4** (`A4_bond_heavy_pathology.md`) — distinct from **Table A.4** above.

A.3 Glide vs. flat allocation — full Epoch 6 grid (percentage differences)

This appendix supplements **§5.7** of the main paper. For each cell, entries are $100 \times$ (glide $-$ flat)/flat (percent) for the paired experiments at the same terminal equity target. **Dynamic ARVA** and **DAGM** (`corridor_governed_dynamic`) are reported. The withdrawal metrics W_{avg}, W_{min}, and W_{max} are aggregated **across rolling windows** as in §5.7 (mean of window averages for W_{avg}; minimum across windows of window minima for W_{min}; maximum across windows of window maxima for W_{max}). Source: `qa_lab/new_epochs/epoch_6/runs/`. **Figures** for the headline Table 2 comparison and DAGM heatmaps appear in **Appendix C**.

Table A.5 — EUR (EUR_1999_2025, 20-year horizon, eight rolling windows): glide vs. flat

Years to target	Terminal equity	ARVA Δ% W_{avg}	ARVA Δ% W_{min}	ARVA Δ% W_{max}	DAGM Δ% W_{avg}	DAGM Δ% W_{min}	DAGM Δ% W_{max}
5y	40%	2.1	10.0	-10.4	1.2	13.6	-1.7
5y	60%	4.9	18.4	-6.1	2.6	30.4	-2.9
5y	80%	8.6	28.4	-0.7	5.1	13.8	4.7
10y	40%	2.3	11.2	-10.6	1.4	13.6	-0.9
10y	60%	5.9	24.2	-5.8	3.6	31.9	0.2
10y	80%	10.9	39.2	-0.5	7.5	29.3	3.7
20y	40%	0.8	14.1	-12.6	-0.2	13.6	-7.0
20y	60%	3.0	36.2	-15.8	0.5	31.9	-11.8
20y	80%	6.6	65.1	-15.2	2.9	31.9	-12.7
Mean (9 cells)		**5.0**	**27.4**	**-8.6**	**2.7**	**23.3**	**-3.2**

The bottom row is the arithmetic mean across the nine cells and matches the EUR block in §5.7, Table 2 of the main paper.

Table A.6 — USD (USD_1928_2025, 30-year horizon, 69 rolling windows): glide vs. flat

Years to target	Terminal equity	ARVA Δ% W_{avg}	ARVA Δ% W_{min}	ARVA Δ% W_{max}	DAGM Δ% W_{avg}	DAGM Δ% W_{min}	DAGM Δ% W_{max}
5y	40%	-0.2	6.2	4.0	-1.6	-3.6	4.6
5y	60%	0.7	3.1	10.5	-2.7	-2.2	16.8
5y	80%	2.8	7.2	17.2	-3.0	-1.5	5.2
10y	40%	-2.8	3.4	2.2	-4.2	-0.6	2.6
10y	60%	-4.4	1.9	6.9	-7.8	-0.4	12.6
10y	80%	-4.9	5.7	11.6	-10.5	-2.2	-2.3
20y	40%	-6.6	-2.0	-3.4	-8.0	-2.2	-2.2
20y	60%	-11.7	0.0	-5.4	-15.0	-0.9	-0.8
20y	80%	-15.6	4.3	-6.6	-21.0	-1.9	-18.7
Mean (3 cells, 5y only)		**1.1**	**5.5**	**10.6**	**-2.4**	**-2.4**	**8.9**
Mean (9 cells)		**-4.7**	**3.3**	**4.1**	**-8.2**	**-1.7**	**2.0**

The row **Mean (3 cells, 5y only)** reproduces the USD aggregates in §5.7, Table 2 of the main paper (5-year glide to target, three terminal targets). The row **Mean (9 cells)** averages over all three glide speeds; longer transitions (notably 20y to target) are associated with **negative** changes in W_{avg} for both policies in this USD sample, so the all-cell mean differs markedly from the 5y-only headline. §5.7 therefore reports the **5y USD** block for direct comparability with the §5.7 Table 2 headline (5y USD, three terminal targets); the nine-cell USD average is a complementary summary.

> **Appendix A.4 (Paper v2.1 supplement):** Edit **this file** after new Epoch/Lab reruns. **Paper v2.1** cites this appendix where USD **20/80** tail risk is discussed (§5.3, §8.2); the main paper PDFs do **not** embed this section body. **Not Table A.4** (Epoch 5C four-asset win counts in §A.2 of `A_empirical.md`).

Appendix A.4 — Bond-heavy USD allocation: timing of exhaustion, stagflation vignette, and reading ARVA-vs-DAGM floors

A.4.1 Purpose

This appendix **does not introduce new empirical tables** carried in §5 or in the appendix supplement grids. Instead it **ties together mechanics** discussed in §5.2–5.4 and §8.2 for the **worst behaved two-asset sleeve of the Epoch**

matrix, the **USD 20% equity / 80% nominal bond mix** rolled out as Epoch 2 cases `usd_2080_rolling_{20|30}y_corridor_governed`. Readers may treat §5 narrative as authoritative for **scores and wins**; Appendix A.4 clarifies three questions that recur in policy discussion:

1. When ARVA wins on **within-window** $W_{\min}^{\mathbf{real}}$ in this sleeve, **does DAGM literally "hit zero" midway through the horizon**, or is the stress mostly a **consumption-real** deterioration?
2. What **economic mechanism** aligns with the **worst YoY shocks** documented in Epoch 3 for this allocation?
3. How should one interpret **different terminal-wealth profiles** alongside **floors**, without falsely equating $W_{\min}$ with "household safety" on every path?

Reproducibility: year-by-year ledgers and summary rows are in `qa_lab/new_epochs/epoch_2/runs/usd_2080_` and `…__summary.json` (and the 20-year companions). The script `qa_lab/new_epochs/analyze_usd_2080_dag` regenerates ranked windows and optional head/tail traces.

A.4.2 Definitions: "early" vs "final-year" exhaustion

For each rolling **start year** and policy track, let $\{\texttt{capital\_end}_\tau\}$ denote end-of-year capital after withdrawals and simulated returns within the simulated horizon $\tau \in \{1, \ldots, T\}$ (indexed by `sim_year_index`). Using the same bookkeeping as the Epoch 5 four-asset diagnostics in §5.5.C (**early depletion:** non-positive `capital\_end` in any $\tau < T$; **last-year exhaustion:** $\texttt{capital\_end}_T \leq 0$ but strictly positive $\texttt{capital\_end}_\tau$ for all $\tau < T$).

For implementation we treat `capital_end` ≤ 1 nominal currency unit after rounding as depleted (insensitive to negligible floating residuals).

A.4.3 Empirical decomposition (USD 20/80 Epoch 2, post–Manage CPI-fix batch)

Across **all overlapping rolling windows**:

Horizon	Metric	Pure **ARVA**	**DAGM** (`corridor_governed_dynamic`)
30-year (**69 paths**)	Depletion strictly **before final year** $\tau < T$	**0**	**0**
	Exhaustion **only** in calendar **final simulated year**	**69**	**7** starts: **1928, 1950, 1951, 1952, 1953, 1954, 1955**

Horizon	Metric	Pure **ARVA**	**DAGM** (`corridor_governed_dynamic`)
	Terminal wealth still positive at horizon end	**0**	**62**
20-year (**79 paths**)	Depletion before final year	**0**	**0**
	Exhaustion only in final year	**79**	**8** starts: **1935–1937** and **1961–1965**
	Terminal wealth still positive	**0**	**71**

Reading: In this dataset **neither** policy exhibits **mid-horizon capital blow-ups** for these two equity/bond sleeves: when accounts go to ~zero, it is **at the last simulated year** of the rolling window. The **qualitative difference** is that **ARVA exhausts capital on every path** at this allocation and horizon length in the Lab batch, while **DAGM usually preserves a positive terminal cushion**; only a **small minority** of rolling starts drive DAGM to **final-year** depletion. This **separates "plan failure before horizon"** from **"high spending that ends with no legacy"**—the latter is the **dominant ARVA outcome** here, not a coding accident.

A.4.4 Worked path: `start_year = 1950`, USD 20/80, 30-year (`horizon` ends 1979)

This window `start_year = 1950` is emblematic because (i) it belongs to the **1974 → 1975** cluster underlying the **largest within-window YoY decline in real withdrawals** among historical paths sharing that era, and (ii) among the `corridor_governed_dynamic` runs on **USD 20/80 · 30y**, it is one of **seven** rolling starts $\{1928, 1950, 1951, 1952, 1953, 1954, 1955\}$ where **DAGM's capital sinks to ( )0 only at the horizon's last simulated year**, while `governance_status` **remains GREEN** through the stagflation spike (definitions align with §5.5.C's "Early depl." versus "Last-yr depl." bookkeeping). Across the full **69** DAGM paths, **62** still carry **positive** terminal cushions; juxtaposing $W_{\min}^{\text{real}}$ advantage for ARVA without acknowledging **capital residue** invites mis-reading.

- **1973–1974:** `ARVA_vs_GK_dominance = gk_lower`. The corridor still has **thousands of nominal dollars** of width between **GK** (lower bound) and **ARVA** (upper bound). **High CPI** and weak real returns **compress** that width rapidly (from ~15.7k nominal width in 1973 to ~5.1k in 1974).

- **1975:** Dominance flips to `arva_lower`, **corridor** `inverted = true`, and **nominal width collapses to ~773** with **withdrawal pinned between nearly coincident GK/ARVA anchors (~76.6k–77.4k nominally)** while **inflation stays elevated**. **Real withdrawals**, CPI-deflated year-by-year exactly as Epoch 3 stability exports, **fall about 14 % YoY**—the steepest single-year contraction on this path—even though `governance_status = GREEN`.
- `w_dagm_withdrawal_source` remains `manage_w_target_by_status`; the episode is thus **not** a badge-red override but **tight-binding geometry** plus inflation.

Interpretation aligned with §5.3: the stressor is primarily a **living-standard shock in real units** conditioned on a **bond-heavy portfolio** encountering **nominal-anchor squeeze**, not premature badge-driven liquidation in this vignette.

A.4.5 Calendar clustering of worst YoY real cuts (30-year DAGM, all starts)

Ranking, for **each rolling window independently**, the **largest YoY percentage decline** in real withdrawal (constructed as in Epoch 3 stability diagnostics from `yearly_rows`), yields modal **calendar landing years**:

Landing calendar year τ of worst cut	Paths (share of **69**)	Representative adjacent transition
1975	**21 / 69**	1974 → 1975
1995	**14 / 69**	1994 → 1995
1947	**10 / 69**	1946 → 1947
1942	**8 / 69**	1941 → 1942
1981	**6 / 69**	1980 → 1981
1970	**6 / 69**	1969 → 1970

The multiplicity on **1975** is **economic concentration**, not RNG: many overlapping 30-year paths share the stagflation-era transition. **BADGE frequency note:** Across the entire `usd_2080` · 30y file, `RED` states on DAGM arise only on **early-1930s** Depression ledgers—not on **1975** observations—underscoring that **risk to lifestyle** may appear while **risk-state remains GREEN**.

A.4.6 Asset-allocation takeaway (design, not calibrated policy table)

Empirical averages in §5.2 favour **DAGM** on corridor metrics for most allocations. For **bond-heavy USD**, Appendix A.4 motivates **dual levers**:

1. **Allocation / glide (§5.6 template):** Reducing simultaneous exposure to **inflation shocks** hitting both **capital** and **nominal corridors** dominates micro-tuning thresholds when the pathology is structural.

2. **Governance / corridor engineering (future sandbox):** When **GK** and **DARVA/ARVA** nominal anchors mechanically **collapse together**, widen or **inflation-link** discretionary slack or escalate control when `corridor_width < \varepsilon`—but any change must be re-scored across **Epoch 2's full sleeve matrix**, not judged on 20/80 alone.

Readers should refuse the shortcut "**ARVA always safer**": **floors $\neq$ exhaustion timing $\neq$ terminal wealth $\neq$ welfare**, as summarized in **§A.4.3** above.

A.4.7 Supplement merging

Included in `DAGM_v2.1_appendix_supplement.pdf` (`scripts/build_appendix_supplement_v21_pdf.py`). This file is the authoritative **Appendix A.4** source for Paper v2.1.

> **Appendix A.5 (Paper v2.1 supplement):** Edit **this file** after new Epoch 11 Lab batches. **Paper v2.1** §8.1 cites this appendix qualitatively; multi-policy tables are in **Appendix A.5b** (§7, Table 8.1). **Not Table A.5** (Epoch 6 glide grid in §A.3 of `A_empirical.md`) and **not Table A.5b.1** (multi-policy grid in `A5b_legacy_policy_compare.md`). The full section body is **not** duplicated in `Paper_v2.1_public.md`.

Appendix A.5 — Legacy share sensitivity and the life-style vs. bequest trade-off (Epoch 11)

A.5.1 Purpose

Contribution **(d)** in §1.3 states that DAGM models a **legacy reserve** ring-fenced from the capital amortized through DARVA, so ordinary spending is anchored on **spendable** wealth while bequest intentions remain explicit. This appendix **quantifies** how that design behaves when the household raises the **initial legacy share** (fraction of plan capital reserved for CPI-indexed legacy from creation) while the **corridor-governed (DAGM)** track and Sandbox-default reference parameters are held fixed.

The exercise is **not** a new §5 "epoch" in the sense of ranking DAGM against static rules. It is a **sensitivity grid** on one governance lever: {0, 10, ..., 70}% on three historical sleeves already used elsewhere in the Lab matrix.

Reproducibility: `python qa_lab/experiments/run_legacy_agreed_sweep.py` (optional `--sync-epoch11`). Regenerate tables: `python qa_lab/new_epochs/epoch_11/build_legacy_sweep` Archived batch used below: `qa_lab/new_epochs/epoch_11/runs/legacy_agreed_sweep_20260515_192045Z/`

A.5.2 Design

- **Policy track in tables:** `corridor_governed_dynamic` (DAGM) only.
- **Legacy:** `legacy_mode: cpi_indexed_from_creation`; = `legacy_share_lambda` at plan inception.

- **Sleeves:** (1) EUR 60/40, 20-year rolling (`eur_6040_rolling_20y_corridor_governed`); (2) USD 60/40, 30-year rolling; (3) USD four-asset Epoch-5 menu (eq 60%, 30y).
- **Metrics:** Same rolling convention as §5.2 — each cell is the **mean across rolling start windows** of a window-level statistic (not a global worst start across the full sample).

Year-by-year trade-off (product reading). At each annual review the engine refreshes spendable capital, corridor bounds, governance colour, and the selected withdrawal. A higher **shrinks** the spendable base available for amortization and reference scaling **from year one** while **preserving** (and CPI-indexing) the legacy slice. The household therefore sees, **path by path**, how a stronger bequest commitment reallocates outcomes between **sustainable lifestyle cash flows** (withdrawal level, corridor position, stress colours) and **residual capacity at horizon end** (terminal wealth on the simulated path). DAGM does not collapse that tension into a single headline rate; the tables below summarize how aggregate windows move when shifts.

A.5.3 Results (DAGM track)

Units: Lab export scale (thousands; comma-separated in grids). `C_fin_real` = mean across windows of **terminal wealth, real** (`terminal_wealth_real` in `*_summary.json`).

EUR 60/40, 20y — 8 rolling windows

lambda%	W_avg_real	W_min_real	W_max_real	C_fin_real	green%	amber%	red%	super%
0	66,979	44,358	116,075	68,226	88.1	8.8	3.1	0.0
10	65,543	40,004	132,641	162,124	88.1	8.1	3.8	0.0
20	64,106	35,649	149,208	256,021	88.1	8.1	3.8	0.0
30	62,657	31,666	164,863	348,804	88.1	7.5	3.8	0.6
40	61,336	26,832	181,187	441,780	88.1	5.6	4.4	1.9
50	59,940	22,989	194,129	528,751	88.1	5.0	3.8	3.1
70	57,451	12,570	220,535	700,586	88.8	2.5	3.1	5.6

USD 60/40, 30y — 69 rolling windows

lambda%	W_avg_real	W_min_real	W_max_real	C_fin_real	green%	amber%	red%	super%
0	72,016	41,885	148,311	101,204	95.8	3.0	0.8	0.4
10	73,618	39,443	168,568	159,461	96.1	2.7	0.7	0.5
20	75,164	36,410	189,540	217,136	96.4	2.4	0.6	0.6
30	76,726	32,810	210,037	274,220	96.6	2.2	0.5	0.7
40	78,422	28,828	232,020	332,345	96.7	2.1	0.4	0.8
50	80,150	24,343	255,813	393,037	97.0	1.7	0.4	0.9
70	84,125	14,371	312,508	524,241	97.4	1.0	0.5	1.1

USD 4-asset eq60, 30y — 69 rolling windows

lambda%	W_avg_real	W_min_real	W_max_real	C_fin_real	green%	amber%	red%	super%
0	71,667	43,920	153,319	110,047	96.5	2.5	0.6	0.4
10	73,239	41,616	175,550	169,596	96.7	2.4	0.5	0.5
20	74,770	38,598	196,576	227,642	96.8	2.2	0.4	0.6
30	76,351	34,538	218,244	286,367	96.9	2.1	0.4	0.6
40	77,956	30,040	239,027	343,425	97.1	1.7	0.5	0.7
50	79,669	25,135	261,874	403,069	97.2	1.5	0.4	0.9
70	83,729	14,740	319,029	535,046	97.6	0.7	0.6	1.1

`capital_depletion_flag` is **false** for all listed DAGM windows in this batch.

A.5.4 Interpretation

Common structure. As rises, the mean of within-window **minimum** real withdrawals (**W_min_real**) **falls** in every sleeve: the typical **floor** of the cash-flow path weakens when more wealth is cordoned off at inception. Mean **maximum** real withdrawals (**W_max_real**) and mean **terminal wealth (real) rise** — more upside on the spending path and much larger residual balances at horizon end on the Lab scale. That joint pattern is the empirical face of the **life-style vs. bequest** tension made explicit in the engine.

EUR vs USD on average spending. On the **short EUR** sample, mean **W_avg_real declines** with (about 67k → 57k at 70%). On the **longer USD** sleeves, mean **W_avg_real increases** with . The ranking of average withdrawals is therefore **not universal** across markets and horizon length; the **floor deterioration** and **terminal residual gain** are the more stable messages.

Governance colours. Mean GREEN frequency moves only modestly on EUR; on USD it edges up slightly as rises while SUPER_RED share inches up at the highest . The episode is primarily **cash-flow and residual reallocation**, not a wholesale shift in badge mix.

Use in the main text. §8.1 cites this appendix **qualitatively** only. Cross-policy comparison at fixed steps is in **Appendix A.5b** (§7, Table 8.1). The grids here are **not** a welfare ranking of and do not recommend a legacy share.

A.5.5 Limitations

- **EUR** has only **eight** overlapping 20-year windows; treat magnitudes as illustrative.
- Terminal wealth is an **accounting end-state** on the simulated horizon, not a tax-aware bequest utility.
- Results are conditional on **CPI-indexed-from-creation** legacy and the agreed Sandbox reference defaults in the sweep script.

Appendix A.5b (Paper v2.1 supplement): Edit **this file** after new Epoch 11 Lab batches. **Paper v2.1** §7 cites this appendix; the full section body is **not** duplicated in `Paper_v2.1_public.md`. **Not Appendix A.5** (DAGM-only sweep in `A5_legacy_lambda_sensitivity.md`) and **not Table A.5** (Epoch 6 glide grid in §A.3 of `A_empirical.md`).

Appendix A.5b — DAGM with legacy vs. benchmark policies (historical rolling)

Source: Epoch 11 · `legacy_agreed_sweep_20260515_192045Z` (+ `legacy_sweep_eq50_4asset` for Table A.5b.3) · `summary.json` **Dataset:** USD 1928–2025 · $T = 30$ years · $n = 69$ rolling windows **Initial capital:** $C_0 = 1{,}000{,}000$ · Values in real terms (CPI-deflated)

Naming. Appendix A.5b holds the multi-policy legacy comparison tables below (**Table A.5b.1–A.5b.3**). **Table 8.1** in the main paper (§7.2) is the same grid as **Table A.5b.1** (identical columns and numbers). **Appendix A.5** is a separate DAGM-only sensitivity block (§8.1). **Table A.5** in §A.3 is the unrelated Epoch 6 glide-vs-flat grid.

Aggregation (aligned with §5.2 and Table 8.1). Withdrawal columns $W^{\mathrm{real}}_{\mathrm{avg}}$, $W^{\mathrm{real}}_{\mathrm{min}}$, and $W^{\mathrm{real}}_{\mathrm{max}}$ are **means across rolling start windows** of each window's within-path summary (not global extrema across all windows and years). **Total final** is mean `terminal_wealth_real` per window. **Residual spendable** is mean `terminal_wealth_real` $- \lambda C_0$ per window (DAGM rows with $\lambda > 0$ only; for benchmarks at $\lambda = 0$, residual spendable equals total final).

Publication language. All table headers and metric legends in this section are **English** for SSRN release (aligned with Table 8.1 in the main paper). Legacy v2.0 Italian draft labels and internal product wording are **not** used. Framework name: **Dynamic Allocation and Governance Model (DAGM)**.

Column legend (all tables). Lab export field names in parentheses.

- **Policy / scenario** — DAGM corridor-governed track at the stated legacy share λ (first row: $\lambda = 0\%$, labelled **DAGM**); ARVA, GK, and SWR on full C_0 with no bequest mechanism.
- $W^{\mathrm{real}}_{\mathrm{avg}}$ (`W_avg_real`) — mean real annual withdrawal; §5.2 window-mean convention (same header as Table 8.1).
- $W^{\mathrm{real}}_{\mathrm{min}}$ (`W_min_real`) — mean of each window's minimum real withdrawal.
- $W^{\mathrm{real}}_{\mathrm{max}}$ (`W_max_real`) — mean of each window's maximum real withdrawal.
- **Residual spendable** — for DAGM with $\lambda > 0$: mean terminal wealth minus the real legacy target λC_0. Parentheses: mean terminal wealth falls short of λC_0. — for ARVA (terminal wealth = 0 by actuarial design). For GK/SWR at $\lambda = 0$, equals **Total final**.
- **Total final** — mean real terminal wealth across windows.

Table A.5b.1 — USD 60/40 mix ***(reproduced as Table 8.1 in §7.2)***

Policy / scenario	$W^{\text{real}}_{\text{avg}}$	$W^{\text{real}}_{\text{min}}$	$W^{\text{real}}_{\text{max}}$	Residual spendable	Total final
DAGM	**71,781**	42,804	143,661	90,862	**90,862**
DAGM $\lambda =$ 10%	**73,320**	40,038	162,102	48,028	**148,028**
DAGM $\lambda =$ 20%	**74,892**	37,093	183,421	7,201	**207,201**
DAGM $\lambda =$ 30%	**76,526**	33,471	204,853	(34,675)	**265,325**
DAGM $\lambda =$ 40%	**78,143**	29,166	226,643	(76,219)	**323,781**
DAGM $\lambda =$ 50%	**79,860**	24,469	249,043	(117,457)	**382,543**
ARVA	67,005	41,990	96,924	—	—
GK	53,489	38,410	69,611	1,098,110	**1,098,110**
SWR	40,221	38,350	43,823	1,789,910	**1,789,910**

Table A.5b.2 — USD four-asset 60/20/10/10 (equity/bond/linker/gold)

Policy / scenario	$W^{\text{real}}_{\text{avg}}$	$W^{\text{real}}_{\text{min}}$	$W^{\text{real}}_{\text{max}}$	Residual spendable	Total final
DAGM	**71,326**	44,977	146,046	98,479	**98,479**
DAGM $\lambda =$ 10%	**72,863**	42,321	167,821	58,056	**158,056**
DAGM $\lambda =$ 20%	**74,504**	38,948	189,947	17,300	**217,300**
DAGM $\lambda =$ 30%	**76,229**	34,622	213,013	(22,512)	**277,488**

Policy / scenario	$W_{\text{avg}}^{\text{real}}$	$W_{\text{min}}^{\text{real}}$	$W_{\text{max}}^{\text{real}}$	Residual spendable	Total final
DAGM $\lambda =$ 40%	**77,799**	30,141	234,480	(64,185)	**335,815**
DAGM $\lambda =$ 50%	**79,457**	25,224	257,173	(104,373)	**395,627**
ARVA	66,738	45,538	92,804	—	—
GK	52,600	39,119	67,668	1,141,614	**1,141,614**
SWR	40,212	38,350	43,823	1,876,495	**1,876,495**

Methodological notes

Comparison framing

ARVA, GK, and SWR operate on the full $1{,}000{,}000$ portfolio with no bequest plan. DAGM with $\lambda > 0$ operates on $S_t = W_t - L_t$ (spendable capital) and explicitly ring-fences $C_0 \times \lambda$ as a CPI-indexed legacy reserve. The correct comparison is DAGM versus the others on the same initial C_0: DAGM's $W_{\text{avg}}^{\text{real}}$, although drawn from a smaller base when $\lambda > 0$, exceeds GK and SWR on the full capital, and terminal wealth includes a planned heir share.

Nature of terminal capital in benchmark models

The elevated terminal capital of GK (mean 1,098,110) and SWR (mean 1,789,910) is a structural by-product of conservative withdrawal formulas, not a planned reserve: no share is designated, communicated, or guaranteed. ARVA terminal wealth tends to zero by actuarial construction — no bequest is possible, even accidentally.

λ and mean shortfall

At $\lambda = 30\%$ and $\lambda = 40\%$, mean residual spendable is negative. This does not indicate plan failure, but that in some historical windows (adverse market periods) total capital at horizon end falls slightly short of the real legacy target planned at plan open. Realized legacy nevertheless remains in the order of magnitude of the target. $W_{\text{avg}}^{\text{real}}$ rises with λ because the DAGM corridor on S_t extracts retirement income more efficiently from the available spendable base.

Table A.5b.3 — USD four-asset 50/30/10/10 (equity/bond/linker/gold)

Policy / scenario	$W^{\text{real}}_{\text{avg}}$	$W^{\text{real}}_{\text{min}}$	$W^{\text{real}}_{\text{max}}$	Residual spendable	Total final
DAGM	**65,809**	43,882	129,413	85,221	**85,221**
DAGM $\lambda =$ 10%	**66,934**	41,338	148,911	42,767	**142,767**
DAGM $\lambda =$ 20%	**68,065**	38,350	168,736	417	**200,417**
DAGM $\lambda =$ 30%	**69,257**	34,621	188,757	(42,120)	**257,880**
DAGM $\lambda =$ 40%	**70,403**	30,198	208,224	(85,328)	**314,672**
DAGM $\lambda =$ 50%	**71,571**	25,356	227,469	(129,133)	**370,867**
ARVA	62,550	43,359	84,278	—	—
GK	48,537	38,902	60,441	983,875	**983,875**
SWR	40,212	38,350	43,823	1,494,489	**1,494,489**

Note on high λ and the alternative structure

Values of λ above 40% materially reduce spendable capital S_t, undermining plan performance: the DAGM corridor operates on an ever-smaller base and mean within-window $W^{\text{real}}_{\text{min}}$ declines (e.g. 29k at $\lambda = 40\%$, 25k at $\lambda = 50\%$ in Table A.5b.1). Use of λ above 40% is therefore not indicated in the main retirement plan. Clients with strong bequest intent are better served by a **dedicated inheritance portfolio** — separate account with objectives and allocation aligned to heirs' horizon (typically aggressive equity or growth mix) — leaving the DAGM plan to operate on full spendable capital without reserve constraints.

Generated from: `qa_lab/new_epochs/epoch_11/runs/legacy_agreed_sweep_20260515_192045Z/` *and* `legacy_sweep_eq50_4asset_20260530_051335Z/` *· means across rolling windows ·* `references_use_spendable_capital: true` *·* `legacy_mode: cpi_indexed_from_creation`

Appendix A.6 — Epoch 12bis Path-Depth Validation (Two Allocations)

This appendix supplements **§5.6** of the main paper (Paper v2.1). It publishes the **full policy tables** for the Epoch **12bis** Lab batch: **two** USD allocation menus—**60/40** equity–bond and **50/30/10/10** equity–bond–linker–gold—each tested under **two** sequence designs (1,000-path block bootstrap and **Möbius stress tape** rolling), increasing path depth beyond a conventional rolling panel (~70 windows).

Labeling. This block is **Appendix A.6** (Epoch 12 path-depth). Do not confuse it with **Table A.6** under §A.3 of `A_empirical.md`, which is the Epoch 6 **glide vs. flat** grid cited from **§5.7**.

The batch does **not** re-run chronological historical rolling; that evidence remains in **§5.1**–**§5.5** and Appendices **A.1**–**A.3**.

A.6.1 Experimental protocol

- **Repository path:** `qa_lab/new_epochs/epoch_12/` (May 2026, post–Manage CPI-fix engine, `dagm_alive_refs` off).
- **Plan:** same initial capital and $T = 30$ years as §5.2 (USD dataset).
- **Menus:** **60/40** equity–bond (literature benchmark; linkers excluded from portfolio weights) and **50/30/10/10** equity–bond–linker–gold (linker active from 2004 in historical data).
- **Policies:** DAGM (corridor-governed dynamic), DARVA, GK, SWR, CPW.
- **Metrics:** `lab_policy_w_v1` — per path/window: `aggregate_yearly_path`; across paths/windows: mean of W_{avg}, min of W_{min}, max of W_{max}; depletion flags as in the main text.

Regenerate this appendix:

```
python documentation/Paper_v2.1/appendix/refresh_appendix_a6_epoch12.py
python scripts/build_appendix_supplement_v21_pdf.py --refresh-tables
```

Sources: `qa_lab/new_epochs/epoch_12/runs/mc/reference/*.json`, `runs/mobius/reference/*.json`.

A.6.2 The two sequence regimes

1. **Block bootstrap (Table A.6.1):** 1,000 independent paths from `USD_1928_2025` via 5-year block resampling (seed 42). Tests robustness to **reshuffled** macro segments.
2. **Möbius stress tape (Table A.6.2):** 50 rolling windows on `USD_STRESS_TAPE_10x5_v1` — a **fixed** cyclic chain of crisis blocks (historical bears plus synthetic stress). Tests **sequential** harshness, not path luck.

A.6.3 Depletion columns

Same rule as §5.6 (`classify_plan_ruin` in `qa_lab/batch_runs/lab_track_health.py`):

- `dep_prem`: early plan failure — capital depleted while **fewer than** T planned years were executed (includes truncated paths; e.g. SWR on the Möbius tape 18/30 years).
- `dep_last`: terminal-year depletion after a **full** T-year run. High DARVA `dep_last` rates under bootstrap are often **mechanical** amortization, not early insolvency.

Prior export bug (May 2026): Möbius collectors that counted early depletion only on "completed" windows reported **0/50** for SWR; regenerated references align with MC bootstrap.

A.6.4 Aggregate tables

Table A.6.1 — Block-bootstrap Monte Carlo (5-year blocks) Dataset `USD_1928_2025`, blocks **5y**, horizon **30y**, **1,000** paths, seed **42**. Aggregation `lab_policy_w_v1`. Withdrawals in real units per year (same numeraire as §5.2). `dep_prem` / `dep_last` = paths with early depletion / terminal-year depletion.

Lab reference generated: `2026-05-18T18:45:20.240200+00:00`.

60/40 (E/B) Weights: equity=0.6, bond=0.4, linker=0, gold=0.0.

Policy	W_avg_real	W_min_real	W_max_real	dep_prem	dep_last
DAGM	72,130	9,672	737,872	0/1000	26/1000
DARVA	66,879	7,867	364,414	0/1000	1000/1000
GK	53,134	12,240	299,077	2/1000	3/1000
SWR	35,890	11,081	127,999	32/1000	7/1000
CPW	53,462	7,072	292,134	0/1000	0/1000

DAGM path-level W_{avg} *percentiles: p10=43,366, p50=65,632, p90=108,886.*

50/30/10/10 (E/B/linker/gold) Weights: equity=0.5, bond=0.3, linker=0.1, gold=0.1.

Policy	W_avg_real	W_min_real	W_max_real	dep_prem	dep_last
DAGM	66,865	17,257	548,812	0/1000	13/1000
DARVA	63,307	12,872	324,698	0/1000	1000/1000
GK	49,322	13,545	238,605	0/1000	1/1000
SWR	35,899	11,081	127,999	21/1000	3/1000
CPW	49,620	10,884	250,124	0/1000	0/1000

DAGM path-level W_{avg} *percentiles: p10=44,872, p50=63,166, p90=93,125.*

Table A.6.2 — Möbius stress tape on an extreme bear-alternating sequence (50 rolling windows) Dataset `USD_STRESS_TAPE_10x5_v1` (Möbius stress tape): cyclic **10×5-year** tape, **50** rolling windows, horizon **30y**. Aggregation `lab_policy_w_v1`. Tape composition: `qa_lab/datasets/stress_tape_10x5/README.md`.

Do not compare withdrawal levels to Table A.6.1 (reshuffled history vs Möbius stress tape).

Lab reference generated: `2026-05-19T06:38:35.123192+00:00`.

60/40 (E/B) Weights: equity=0.6, bond=0.4, linker=0, gold=0.

Policy	W_avg_real	W_min_real	W_max_real	dep_prem	dep_last
DAGM	18,187	1,681	61,558	0/50	0/50
DARVA	18,074	1,702	65,888	0/50	0/50
GK	19,032	1,553	57,192	0/50	0/50
SWR	40,306	31,360	55,725	50/50	0/50
CPW	17,632	1,800	64,160	0/50	0/50

DAGM 12k/year comfort threshold: 48.7% of simulated years below; 50/50 windows with at least one year below; worst tape position 36 (window W_{min} *1,681).*

50/30/10/10 (E/B/linker/gold) Weights: equity=0.5, bond=0.3, linker=0.1, gold=0.1.

Policy	W_avg_real	W_min_real	W_max_real	dep_prem	dep_last
DAGM	20,117	3,306	59,459	0/50	0/50
DARVA	19,838	3,317	62,584	0/50	0/50
GK	21,202	3,130	56,448	0/50	0/50
SWR	40,221	31,360	55,725	50/50	0/50
CPW	20,222	3,407	65,914	0/50	0/50

DAGM 12k/year comfort threshold: 38.6% of simulated years below; 50/50 windows with at least one year below; worst tape position 36 (window W_{min} *3,306).*

Table A.6.3 — DAGM path-level percentiles (block bootstrap) Median and tails of **path-level** mean real withdrawal (W_{avg} on each of 1,000 paths).

Menu	p10	p50	p90	Early ruin %	Terminal ruin %
60/40 (E/B)	43,366	65,632	108,886	0.0%	2.6%
50/30/10/10 (E/B/linker/gold)	40,872	63,166	93,125	0.0%	1.3%

Appendix B — Historical market datasets (full construction)

Paper v2.1: Appendix B of `DAGM_v2.1_appendix_supplement.pdf` (built from this file). Main paper Markdown (`Paper_v2.1_*.md`) keeps the §4.5 datasets summary only; full construction is here and in the supplement PDF.

Document: sources and construction rules for the annual return series used in DAGM Sandbox and related analyses.
Currency datasets: USD (1928–2025) and EUR (1999–2025).
Convention: unless stated otherwise, **calendar-year** returns are computed from **December** index levels (**December-to-December**).

Exports: Sandbox — `documentation/DAGM_Dataset_Reference.pdf` (built from this file). **Paper supplement** — `scripts/build_appendix_supplement_v21_pdf.py` embeds this file as Appendix B. **Manage** — same reference content is intended for in-app delivery (parallel to Sandbox).

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1. Reference parameters (models)

These defaults apply to demonstrations and exports unless a scenario specifies otherwise.

ARVA (anchor yields)

Parameter	Value
d_equity	0.065
d_bond	0.030
d_ilbond	0.030
d_gold	0.020

Guyton–Klinger (guardrails)

Setting	Value
GK0	0.5

Setting	Value
g	0.20
s	0.10
H	15
CPI discipline	freeze_cpi = ON
prosperity	ON
capital_preserve	ON
cpi_cap	OFF (stored, inactive)

SWR benchmark

Setting	Value
swr_rate	0.04
swr_cpi_indexed	ON

Nominal vs real

- *Nominal* amounts use the dataset's currency units.
- *Real* amounts deflate using the dataset **inflation rate** column `cpi` (annual rate as a decimal, e.g. `0.021` = 2.1% year-on-year), building a cumulative price index normalized to 1 in the first year of each analysis window.

Column `cpi` (USD and EUR)

- `cpi`: calendar-year inflation rate used for real/nominal conversion in Sandbox and Manage (same numeric convention as the return columns: decimal, not percent).
- `pi`: auxiliary field in the EUR file schema (reserved / model use); `cpi` is the primary series for inflation in reports unless a scenario specifies otherwise.

---

2. USD dataset (1928–2025)

2.1 Core history (1928–2024)

Annual returns for **US equities** (S&P 500 total return, dividends included), **10-year US Treasury total return**, **gold**, and **US inflation** are taken from **Aswath Damodaran (NYU Stern)**, *Historical Returns on Stocks, Bonds and Bills*, data file **histretSP.xls** (stocks, bonds, bills, gold, and CPI).

2.2 Calendar year 2025

The Damodaran release used in this build ends at 2024. Calendar **2025** is appended using independent market sources:

Sleeve	Construction
Equity	S&P 500 **total return** for 2025 from published year-end market summaries.
Bonds	10-year Treasury total return for 2025 proxied by **iShares 7–10 Year Treasury Bond ETF (IEF)** calendar-year total return.
Gold	2025 gold performance from **World Gold Council** annual commentary.
Inflation	US CPI **December-to-December** 2025 from the **BLS** (Consumer Price Index).

2.3 Inflation-linked bonds (TIPS proxy)

- **2004–2025:** `R_ilbond` follows **iShares TIPS Bond ETF (TIP)** total return (price, dividends reinvested).

- **1928–2003:** `R_ilbond` equals the nominal Treasury return series (`R_bond`), as a practical proxy before TIPS and the ETF history.

Interpretation: TIPS did not exist for the full sample; the early period uses the nominal Treasury return as the inflation-linked sleeve proxy.

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3. EUR dataset (1999–2025)

The EUR file provides annual fields including `R_equity`, `R_bond`, `R_ilbond`, `R_gold`, `cpi`, and `pi`. Third-party tools and indices below are cited for transparency; index providers retain their own trademarks and terms of use.

Data tool (index levels): Curvo **Backtest** monthly index exports, **EUR** display where applicable. Curvo presents backtest data aimed at European investors in euro for the indices used here.

3.1 Equity (`R_equity`)

- **Index: MSCI ACWI** (All Country World — developed and emerging markets), as shown in Curvo Backtest with **EUR**.

- **Method:** monthly index levels → **December-to-December** calendar-year return:
  R = (Index at Dec Y ÷ Index at Dec Y−1) − 1 for each year Y

where both December levels exist.

- **Coverage in this build: 1999–2024** from the MSCI ACWI EUR series above.

- **2025:** the published monthly snapshot used for this release did not include a full **December 2025** level for the same Dec/Dec convention; `R_equity` **for 2025** is therefore taken from the **continuous historical EUR dataset** maintained in DAGM (same row as prior releases), so methodology stays consistent for that year without mixing partial-year index data.

3.2 Nominal bonds (`R_bond`)

- **Index (where available): Bloomberg Euro Aggregate Bond** — broad EUR investment-grade bond market, via Curvo Backtest (Euro Aggregate).

- **Method:** monthly levels → **December-to-December** return.

- **Coverage: 2010–2025** from the Euro Aggregate series; **1999–2009** use the **historical EUR dataset** values, because the index history in the snapshot does not support the same Dec/Dec convention for the earliest years.

3.3 Inflation-linked bonds (`R_ilbond`)

- **Index (where available): Bloomberg Barclays Euro Government Inflation-Linked Bond**, via Curvo (example index page).

- **Method:** monthly levels → **December-to-December** return.

- **Coverage: 2006–2024** from the index series; **1999–2005** and **2025** use the **historical EUR dataset** values where the snapshot does not provide the required December levels for a clean calendar-year link.

3.4 Gold (`R_gold`)

- **Reference: Gold bullion in EUR**, Curvo Backtest (Gold bullion, EUR).

- **Method:** monthly levels → **December-to-December** return.

- **Coverage: 1999–2025** from the gold series where the export includes the needed December observations.

3.5 Inflation (`cpi`, `pi`) and other fields (weights, horizons)

- **Source: Eurostat** — euro-area **Harmonised Index of Consumer Prices (HICP)**, all-items, **annual inflation rate** (calendar-year rate as published by Eurostat; stored as a **decimal** in the file, e.g. `0.021` = 2.1%). This matches the §4.5 summary in the main paper.

- **v2build policy:** In `EUR_sandbox_1999_2025_hist_asset_returns_cpi_v2build.csv`, the `cpi` column is **unchanged** from the legacy `EUR_sandbox_1999_2025_hist_asset_returns_cpi.old.` merge (return sleeves were refreshed from Curvo; inflation was not re-sourced), so nominal and real paths stay comparable with prior DAGM releases. The sidecar `EUR_sandbox_1999_2025_hist_asset_returns_cpi_v2build.meta.json` records `source` and `as_of` for audit.

- **`cpi`:** primary inflation series for real/nominal conversion in **Sandbox**, **Manage**, and **Lab** (same column as USD; see §1).

- **`pi`:** auxiliary field in the file schema (reserved / model companion); `cpi` is used for withdrawal and real-capital logic unless a scenario specifies otherwise.

- **Weights** (`w_equity` …), `m`, and horizon `T` are read from the same annual rows as the returns.

---

4. Stress modelling (Manage)

In **sensitivity** and stress tools, nominal bond shocks may use a **representative duration** (default **7 years**) to map interest-rate moves to bond and inflation-linked sleeves. That is a **modelling assumption** for stress, not a year-by-year measure of index duration.

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5. Attribution and disclaimer

- **Damodaran (NYU Stern):** historical US return data.

- **Eurostat:** euro-area **HICP** inflation rates (EUR `cpi` column).

- **Curvo:** access interface to index level series; underlying indices are defined by their **Bloomberg**, **MSCI**, and other providers.

- **MSCI**, **Bloomberg** index names, and other marks are property of their respective owners.

- This document describes **methodology** for the dataset shipped with the application. It is **not** investment advice. Past data do not predict future results. Users remain responsible for compliance with data vendors' terms of use and with applicable regulations.

DAGM dataset reference — Paper v2.1 Appendix B. Subject to updates when indices or calendar-year extensions are refreshed.

Appendix C — Supplementary figures (glide path and life-cycle twins)

This appendix presents charts for the glide vs. flat analysis (§5.7; Appendix A.3 above), the lifecycle **matched-twin** experiments (§6.2–6.3 of the main paper), and the **annual-rebalance couple** results (§6.4). **Numerical authority** remains the tables in the main text and **Tables A.5–A.6** (glide grids) in Appendix A.

Reading notes.

- **Glide vs. flat.** Percentage differences are $100 \times (\text{glide} - \text{flat})/\text{flat}$, with the **§5.7** window-aggregation convention for W_{avg}, W_{min}, and W_{max} (distinct from the mean-of-window-minima convention used for level metrics in §5.2–5.5). The first figure restates the **headline** Table 2 (EUR nine-cell mean; USD 5y-to-target × three terminal weights). The second figure shows the **full nine-cell grid** for **DAGM** only, W_{avg} metric, for EUR and USD (Appendix **Tables A.5–A.6**).
- **USD twins.** The USD twin figure plots means across rolling windows from Table 7.2. Demographics use the **SSA 2022** U.S. period life table with **USD** returns (see §6.2).
- **EUR twins.** The EUR figure shows **fixed minus lifecycle** for mean real W^{avg} only (Table 7.5 gap column). Within each (age, p) cell, twins share the same T and windows; **across** p, horizons and window counts **change** — do not read cross-p differences as identical-path causal effects.
- **Couple plans (§6.4).** Coverage cost is $100 \times (W_0^{\text{med, couple}} - W_0^{\text{med, single}})/W_{0\_}^{\text{med, single}}$ on median initial Phase A withdrawals (M65 single baseline). $\bar{W}_{\text{jump}}$ is the **mean** across rolling windows of the nominal percentage change from the last Phase A withdrawal to the first Phase B withdrawal at actuarial first-death year (GK state carried forward). Source: Epoch 17, `qa_lab/new_epochs/epoch_17/runs/` (Table 7.10).

Source charts (repo). Built by `documentation/charts/build_fig_appendix_*.py`; regenerate with `python documentation/charts/build_appendix_c_assets.py` or `python documentation/charts/build_all_charts.py`.

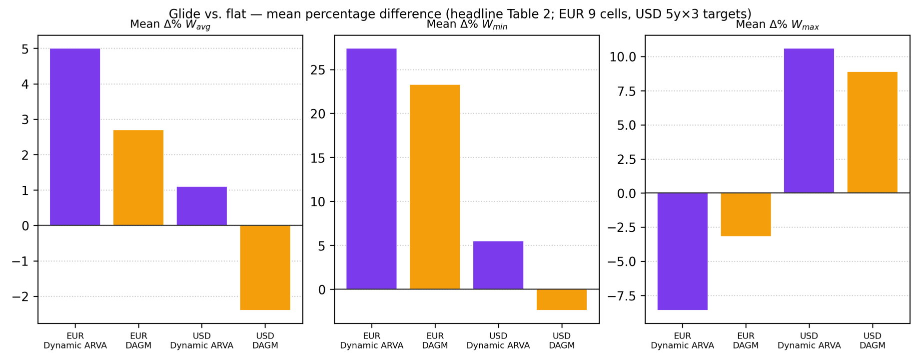

Figure 1: Mean percentage difference glide vs. flat — Table 2 (four region×policy rows; three metrics).

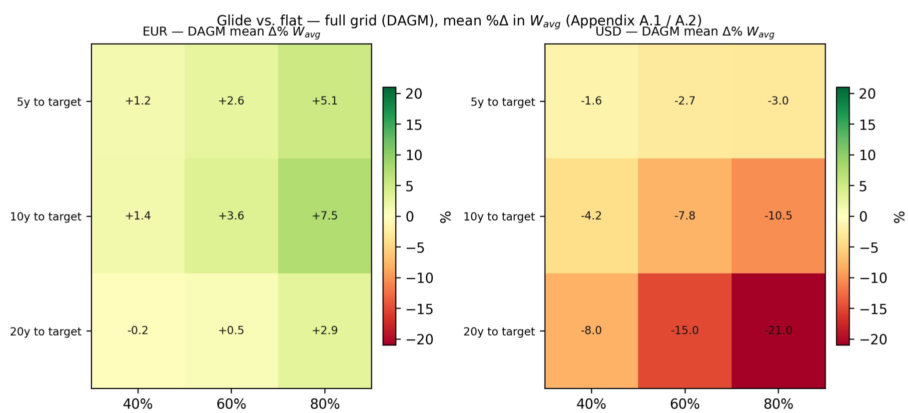

Figure 2: DAGM mean %Δ in W_{avg} — nine-cell grids for EUR and USD (Appendix Tables A.5 / A.6).

C.1 Glide vs. flat — headline and DAGM grid

C.2 Lifecycle matched twins — USD and EUR

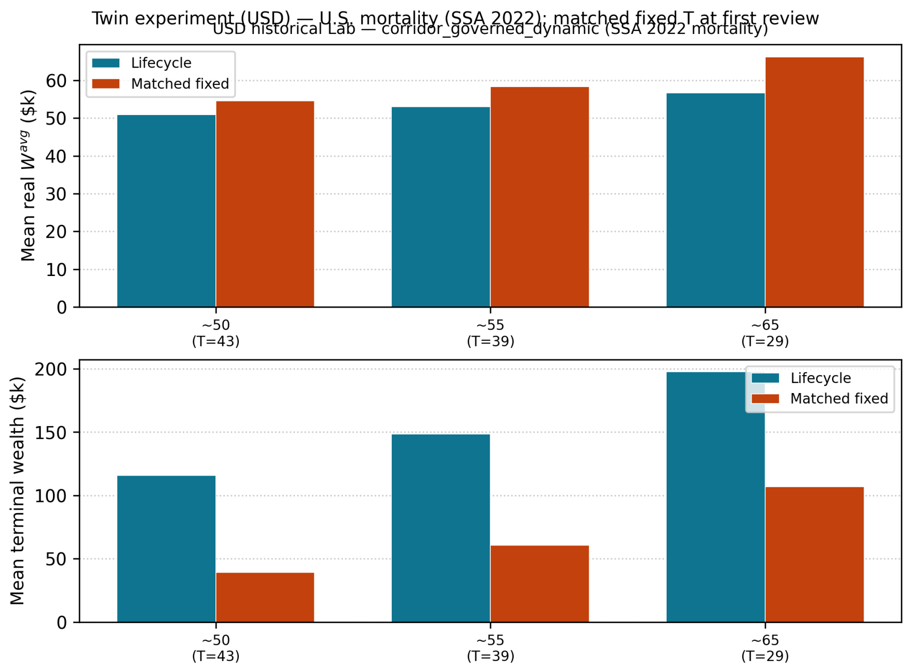

Figure 3: USD twins — mean real W^{avg} and mean terminal wealth (Table 7.2).

C.3 Couple plans — coverage cost and first-death jump (§6.4)

Appendix D — Additional charts (optional)

These assets are generated under `documentation/charts/` for optional use (slides, internal review). They are **not** required to read the main paper PDFs.

Asset	Output (base name)	Build script
Epoch 2 win counts	`fig_epoch2_win_counts`	`build_fig_epoch2_win_counts.py`
Epoch 3 dumbbell	`fig_epoch3_stability_dumbbell`	`build_fig_epoch3_stability_dumbbell.py`
Epoch 3 scatter	`fig_epoch3_stability_scatter`	`build_fig_epoch3_stability_scatter.py`
Epoch 3 preview alternatives	`preview_*.png`	`build_previews_epoch3_alternatives.py`

Asset	Output (base name)	Build script
Epoch 5C four-asset bars	`fig_epoch5c_aggregate_bars`	`build_fig_epoch5c_aggregate_bars.py`

Regenerate everything: `python documentation/charts/build_all_charts.py`.

Source manifest (Paper v2.1 appendix supplement)

Appendix	Source file
A (A.1–A.3, Tables A.2–A.6)	`A_empirical.md` — tables from `refresh_appendix_from_lab.py`
A.4	`A4_bond_heavy_pathology.md`
A.5	`A5_legacy_lambda_sensitivity.md` — tables via `qa_lab/new_epochs/epoch_11/build_legacy_sweep_paper_table.`
A.5b	`A5b_legacy_policy_compare.md` — Epoch 11 multi-policy legacy comparison (§7 Table 8.1)
A.6	`A6_epoch12_validation_matrix.md` — Epoch 12bis MC (1,000 path) + Möbius stress tape (50 windows); `refresh_appendix_a6_epoch12.py`
B	`B_datasets.md`
C	`C_figures.md` — PNG under `documentation/charts/` (`build_appendix_c_assets.py`)
D	`D_additional_charts.md`
Title	`00_title.md`

Main paper (v2.1): `documentation/Paper_v2.1/Paper_v2.1_{public,full_draft,internal}.md` → PDFs via `scripts/build_paper_v21_pdf.py`.

Unified supplement PDF: `documentation/Paper_v2.1/DAGM_v2.1_appendix_supplement.pdf` — build `scripts/build_appendix_supplement_v21_pdf.py` (or `scripts/build_paper_v21_release_pdfs.p`

Lab batches (empirical authority): `qa_lab/new_epochs/epoch_1/` … `epoch_7/`, `epoch_8/` (lifecycle §6), `epoch_9/` (m_comp — internal/full draft only; out of public scope), `epoch_10/`–`epoch_17/` (couple plans — internal), `epoch_11/` (legacy + A.5b), `epoch_12/` (12bis path-depth; Appendix A.6); post–Manage CPI-fix engine (May 2026).

Lifecycle outline: `documentation/lifecycle_horizon.md` (§6 cross-reference).

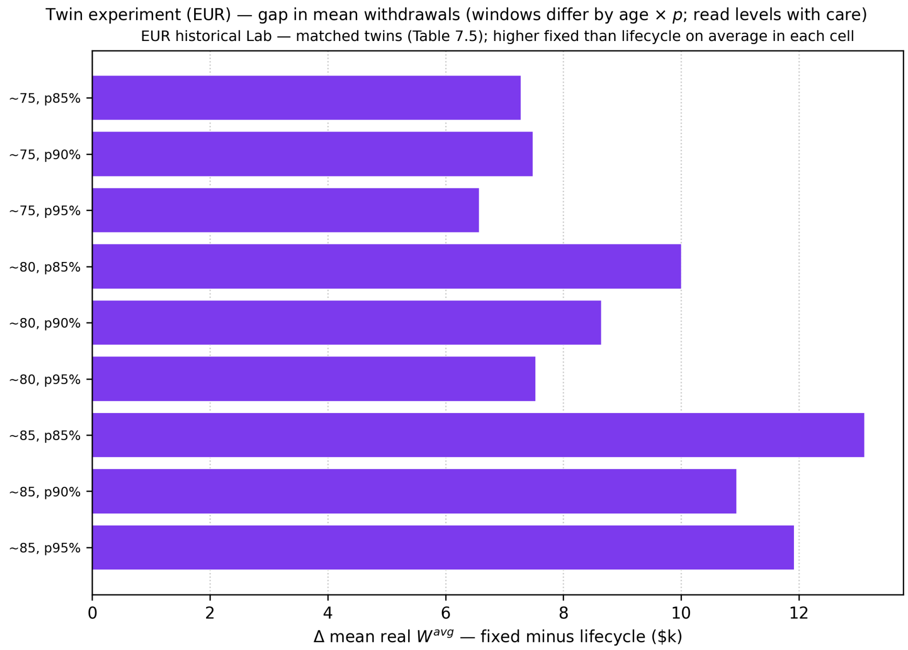

Figure 4: EUR twins — Δ mean real W^{avg} (fixed − lifecycle), nine cells (Table 7.5).

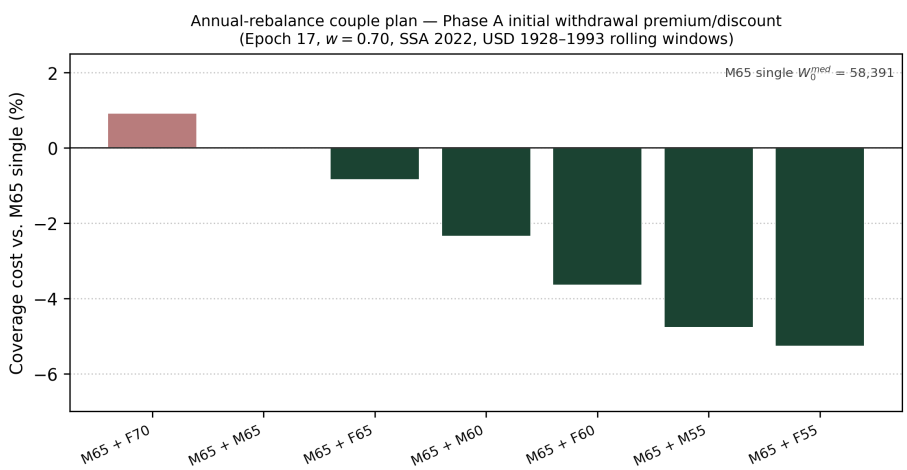

Figure 5: Spousal coverage cost on initial Phase A withdrawal vs. M65 single (Table 7.10).

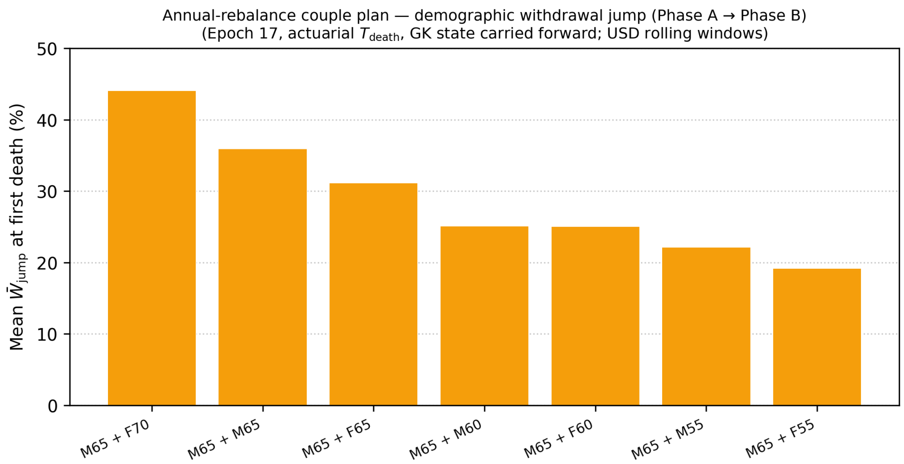

Figure 6: Mean withdrawal jump at first death, annual-rebalance architecture (Table 7.10).

Change log

- **2026-05**: Initial appendix package (v2.0 lineage). Tables **A.2–A.6** from `epoch_1` … `epoch_6` via `refresh_appendix_from_lab.py`.
- **2026-05**: **Appendix A.5** (Epoch 11 legacy sweep).
- **2026-05**: **Appendix A.4** (USD 20/80 pathology).
- **2026-05**: **Appendix B** canonical text in `B_datasets.md`.
- **2026-05**: **Appendix A.6** (Epoch 12bis path-depth).
- **2026-06**: **Paper v2.1 release** — appendix sources canonical under `documentation/Paper_v2.1/appendix/`; supplement PDF `DAGM_v2.1_appendix_supplement.pdf`.
- **2026-06**: **Appendix A.5b** added — multi-policy legacy comparison (`A5b_legacy_policy_compare.md`); cited from §7; metrics aligned to §5.2 window-mean convention.
- **2026-06**: Editorial pass v2.0→v2.1 (citations §8.1 for A.5, §7 for A.5b, §5/§8 for A.4); refresh scripts copied to `Paper_v2.1/appendix/`; `build_appendix_supplement_v21_pdf.py` gains `--refresh-tables`.
- **2026-06**: **Sandbox dataset reference** — `DAGM_Dataset_Reference.pdf` rebuilt from `B_datasets.md` via `scripts/build_dataset_docs_pdf.py`; `DAGM_Dataset_Reference_Client.md` pointer updated to v2.1.
- **2026-06**: **Appendix C.3** — couple-plan figures (Epoch 17 coverage cost and W_{jump}); `epoch17_couple_lab.py` + `build_fig_appendix_couple_*.py`; cited from public §6.4.
- **2026-06**: **Naming** — explicit disambiguation **Table A.4** (Epoch 5C wins) vs **Appendix A.4** (bond-heavy pathology) in `A_empirical.md` and `A4_bond_heavy_pathology.md`.

- **2026-06**: **A.5b** — column headers aligned to Table 8.1 ($W_{\text{avg}}^{\text{real}}$ notation); naming block vs **Appendix A.5 / Table A.5 / Table 8.1**; consolidated column legend before tables.
- **2026-06**: **Language** — A.5b publication-language note + Lab field map (`W_avg_real`, etc.); A.5 grids `C_fin_reale` → `C_fin_real`; Table A.3 cut-frequency label clarified (English).
- **2026-06**: **Release polish** — supplement subtitle without Appendix B parenthetical; public Appendices block lists C.3 (§6.4).
- **2026-06**: **Path audit bundle** — `scripts/build_paper_v21_path_audit_bundle.py` → `release/DAGM_v2.1_path_level_audit_bundle.zip` (summaries + selective `yearly_paths`; no engine/scripts). Reference: `release/PATH_AUDIT_BUNDLE.md`.